

# Improved Partial Coupling for Multi-Phase Flow Solvers

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# Outline

- Numerical approach/coupling methods
- The need for revised partial coupling?
- Study cases
  - Fluidized bed
  - Turbulent open channel
- Conclusions

# Numerical approach (CFD-DEM)

- Volume averaged NS eqs.
  - Coupling through drag & void fractions
  
- Equation of motion
  - Point force models are used
  - Collision model to resolve pp & pw interactions
  
- Coupling techniques
  - Full coupling ( $\varepsilon$  + forces in fluid Eqs.)
  - Partial coupling (only forces in fluid Eqs.,  $\varepsilon=1$ )
    - More stable
    - Faster convergence

## Fluid

$$\frac{\partial(\varepsilon\rho)}{\partial t} + \vec{\nabla} \cdot (\varepsilon\rho\vec{u}) = 0$$

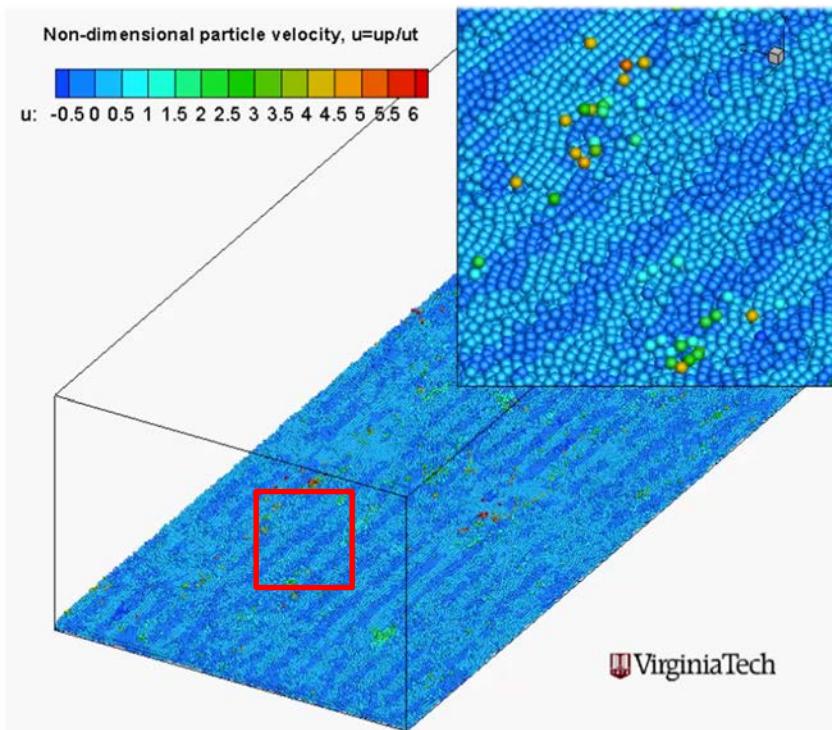
$$\frac{\partial(\varepsilon\rho\vec{u})}{\partial t} + \vec{\nabla} \cdot (\varepsilon\rho\vec{u}\vec{u}) = -\vec{\nabla}p + \vec{\nabla} \cdot (\varepsilon\bar{\tau}) + \rho\vec{g} - \frac{\sum_{N_p} \vec{F}_{drag}}{\forall c}$$

## Dispersed phase

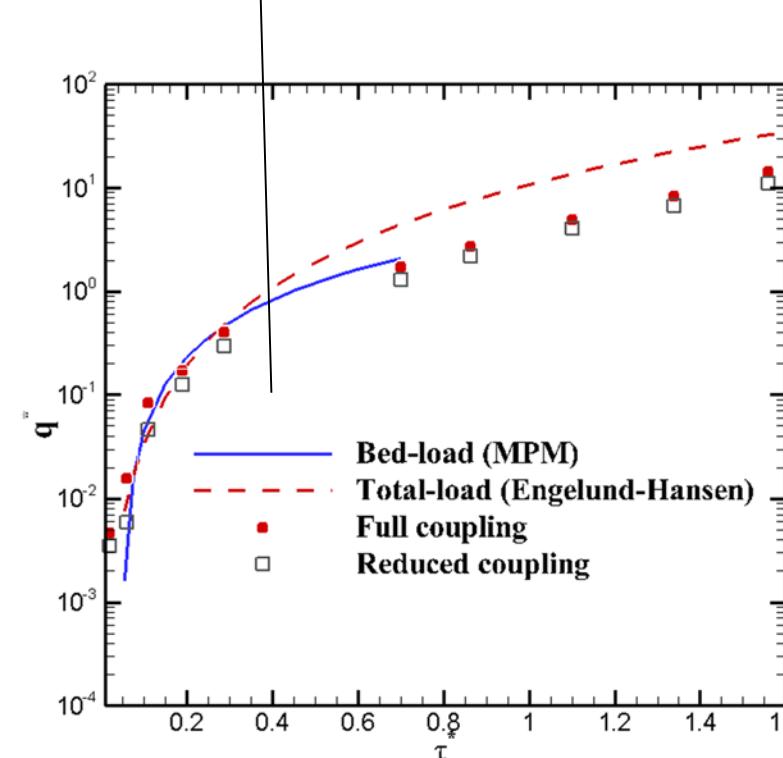
$$m_p \frac{d\vec{u}_{p,i}}{dt} = \vec{F}_{grav,i} + \vec{F}_{drag,i} + \sum \vec{F}_{contact}$$

# Coupling effect in turbulent open channel flow

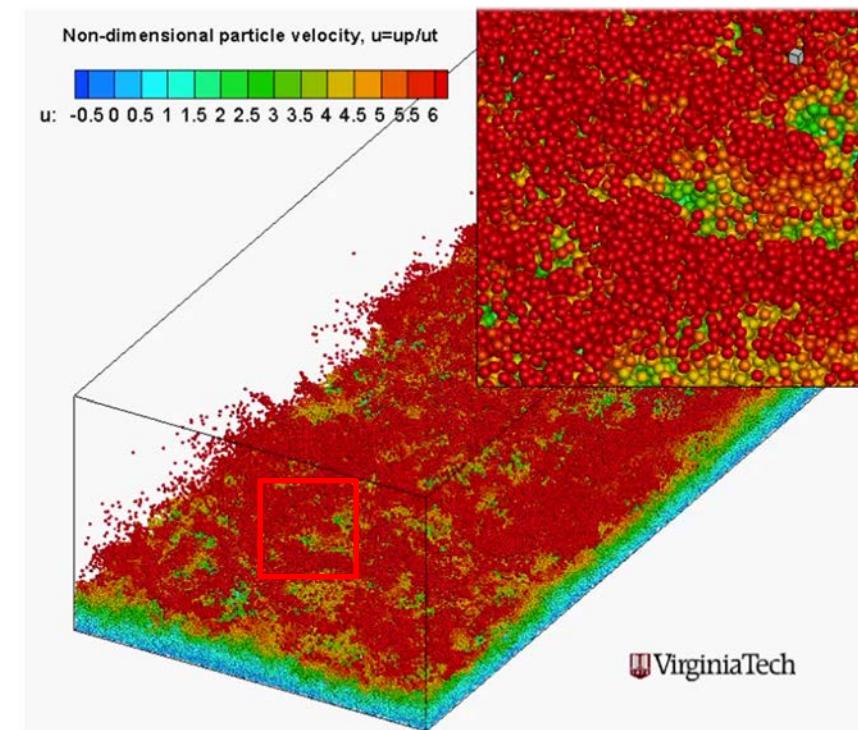
## Bedload



bedload      Suspended load



## Suspended load



$$\text{Einstein number; } q^* \equiv \frac{q_{sx}}{\left( (\rho_s - \rho) g d^3 \right)^{1/2}}$$

$$\text{Shields parameter; } \tau^* \equiv \frac{\rho u_t^2}{(\rho_s - \rho) g d}$$

# Revised partial coupling

- Possible cause of the under-prediction of results is that the interstitial velocity used to calculate the drag becomes smaller than what it should be

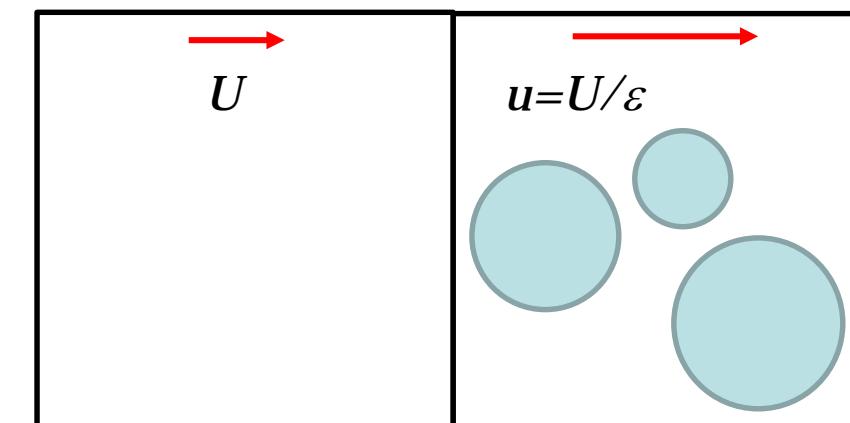
- $$\vec{f}_{Drag} = \frac{\beta}{(1-\varepsilon)\rho_p} (\vec{u} - \vec{u}_p)$$

- A possible remedy is by introducing the effect of void fraction on the interstitial velocity in the drag formulation

$u$  : interstitial fluid velocity

$U$  : Undisturbed/superficial velocity

*Interstitial velocity  $u=U/\varepsilon$*

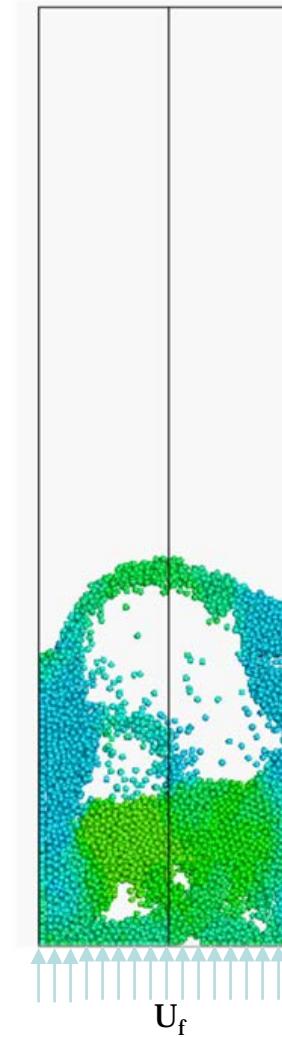


# Case-1: Fluidized bed

- Muller et al. (2009) Experiment

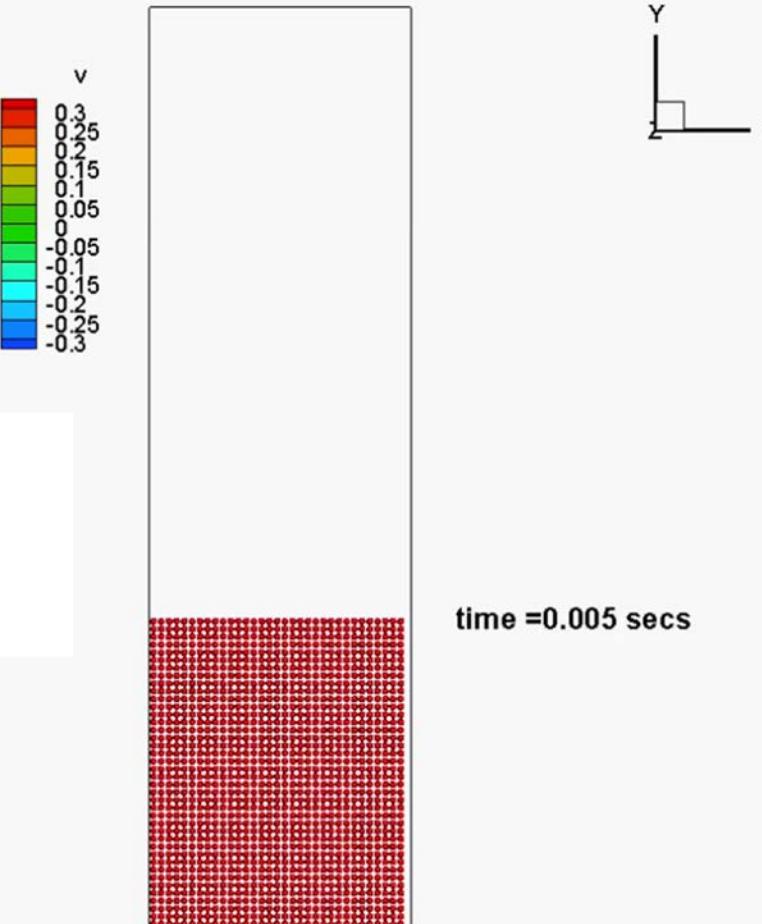
- 44 mm × 160 mm × 10 mm
- $U_f = 0.9 \text{ m/s}$  ( $\sim 3U_{mf}$ )
- 9342 particles

Particles properties (Poppy seeds)	
Diameter	1.2 mm
Density	1000 kg/m <sup>3</sup>
Friction coeff.	0.1
Restitution coeff.	0.98 m/s
Stiffness coeff.	100 N/m
Fluid properties (Air)	
Density	1.205 kg/m <sup>3</sup>
viscosity	$1.8 \times 10^{-5} \text{ kg.m}^{-1}\text{s}^{-1}$



# Fluidized bed: Animation

Full coupling



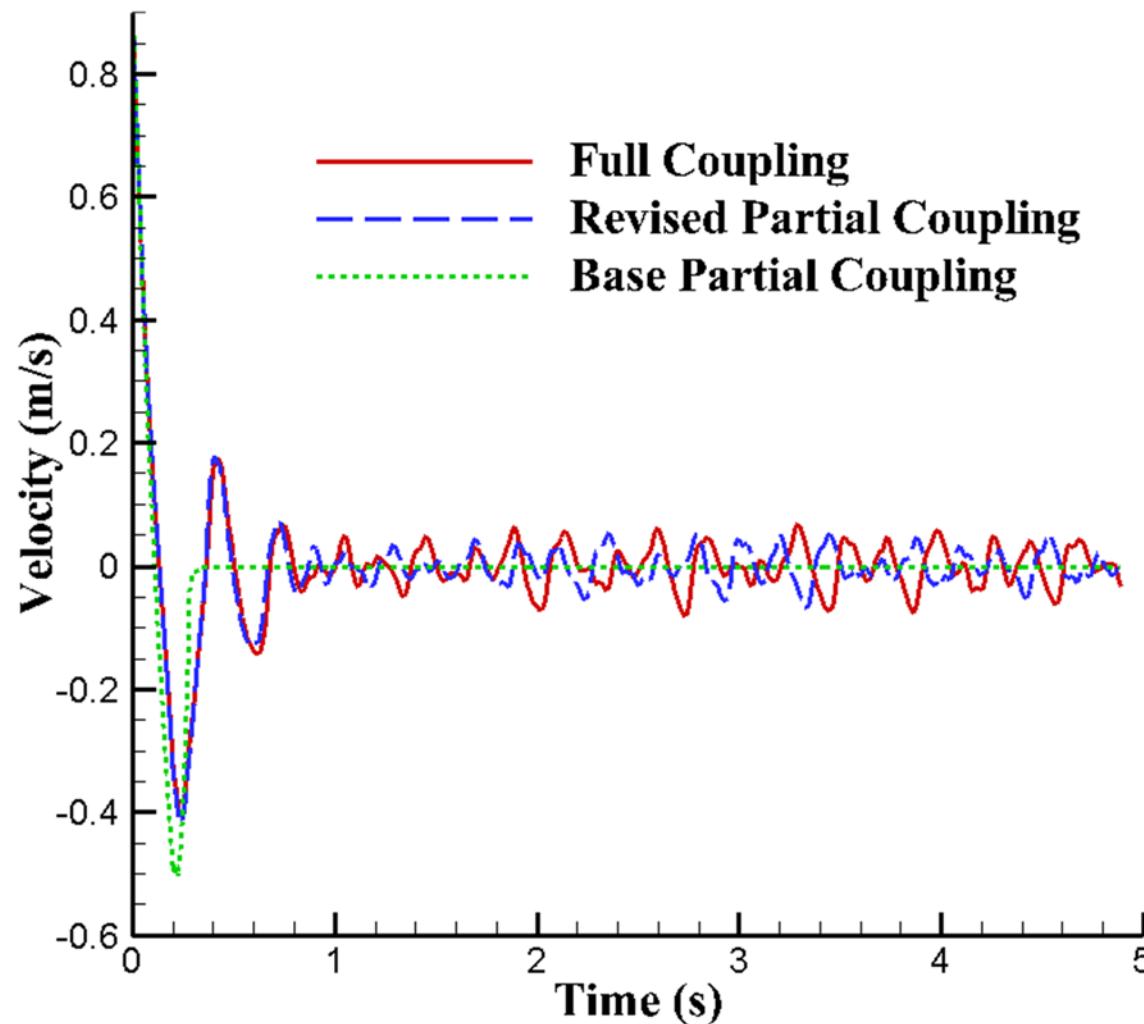
Revised PC



Base PC

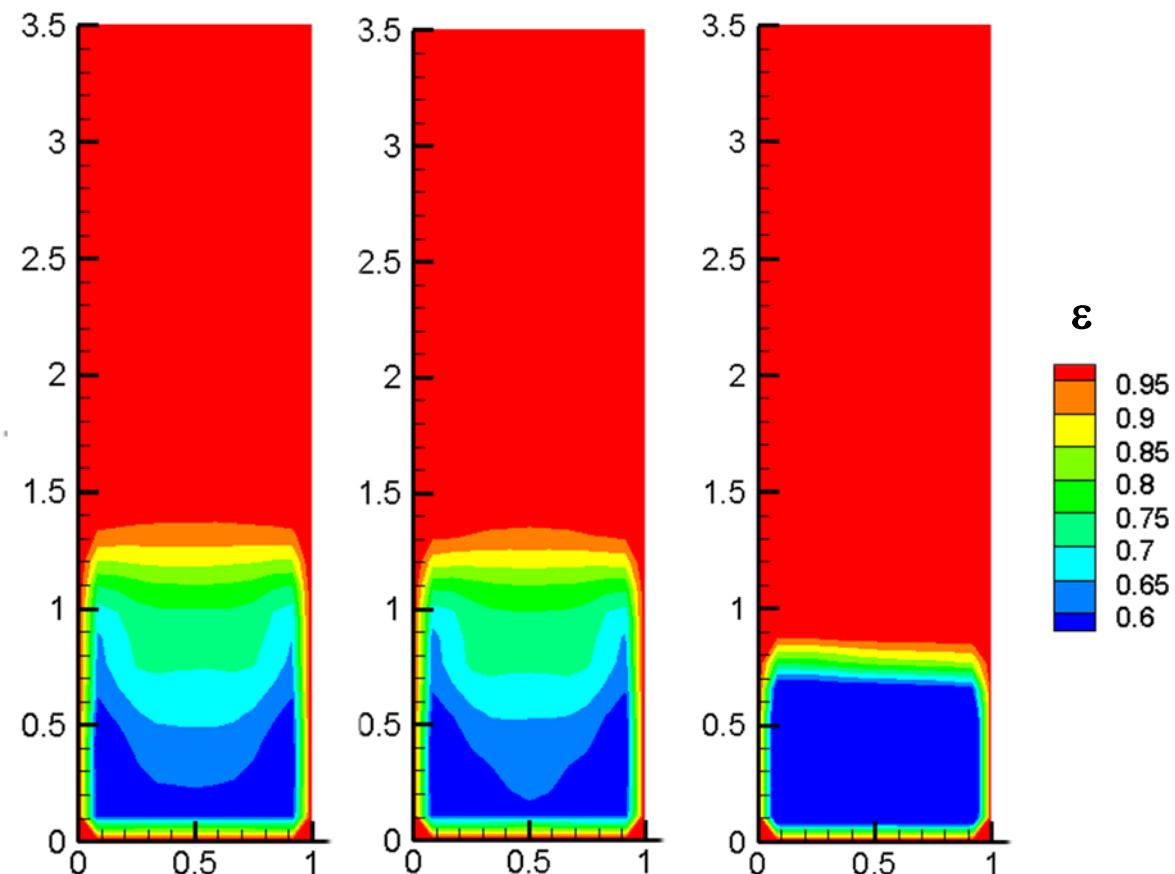


# Fluidized bed: Time series

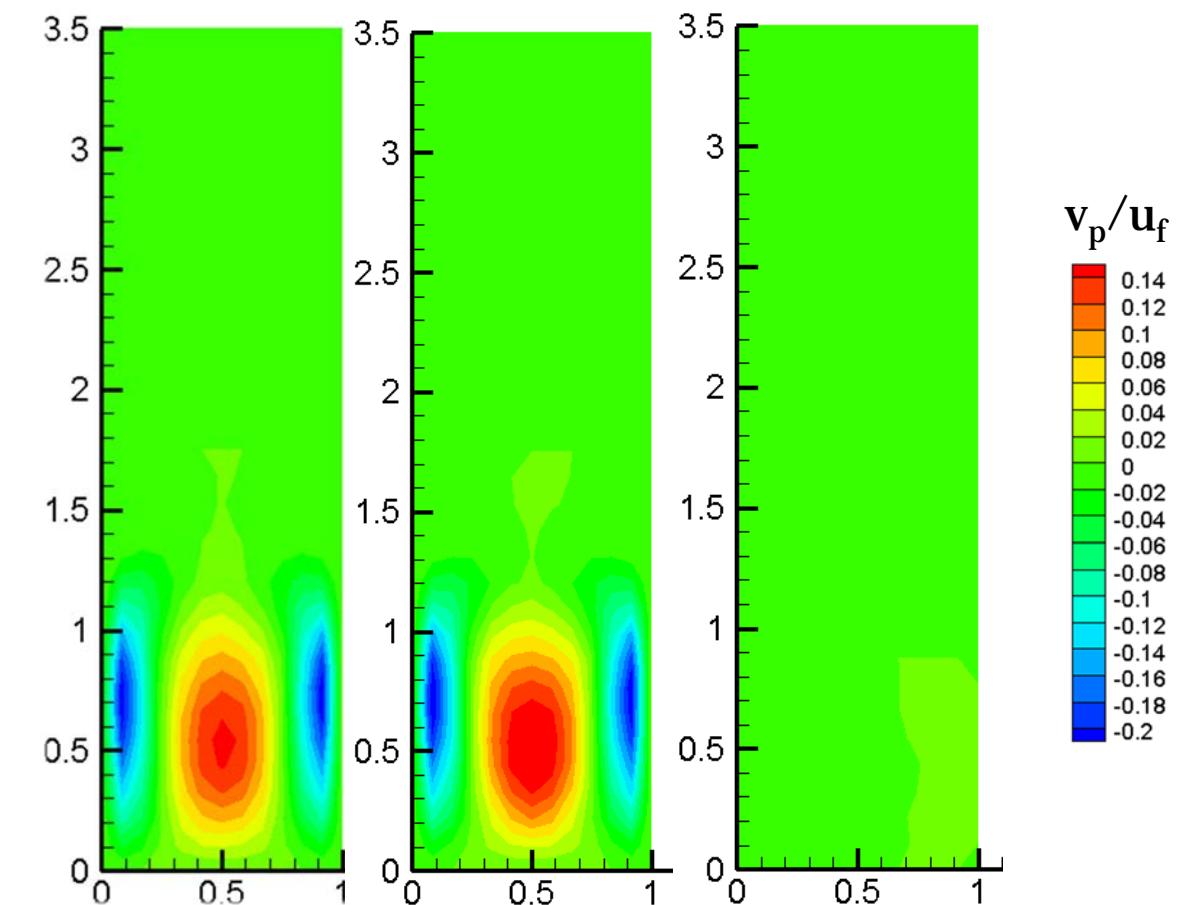


# Fluidized bed: Averaged results

Void fraction



Solids velocity

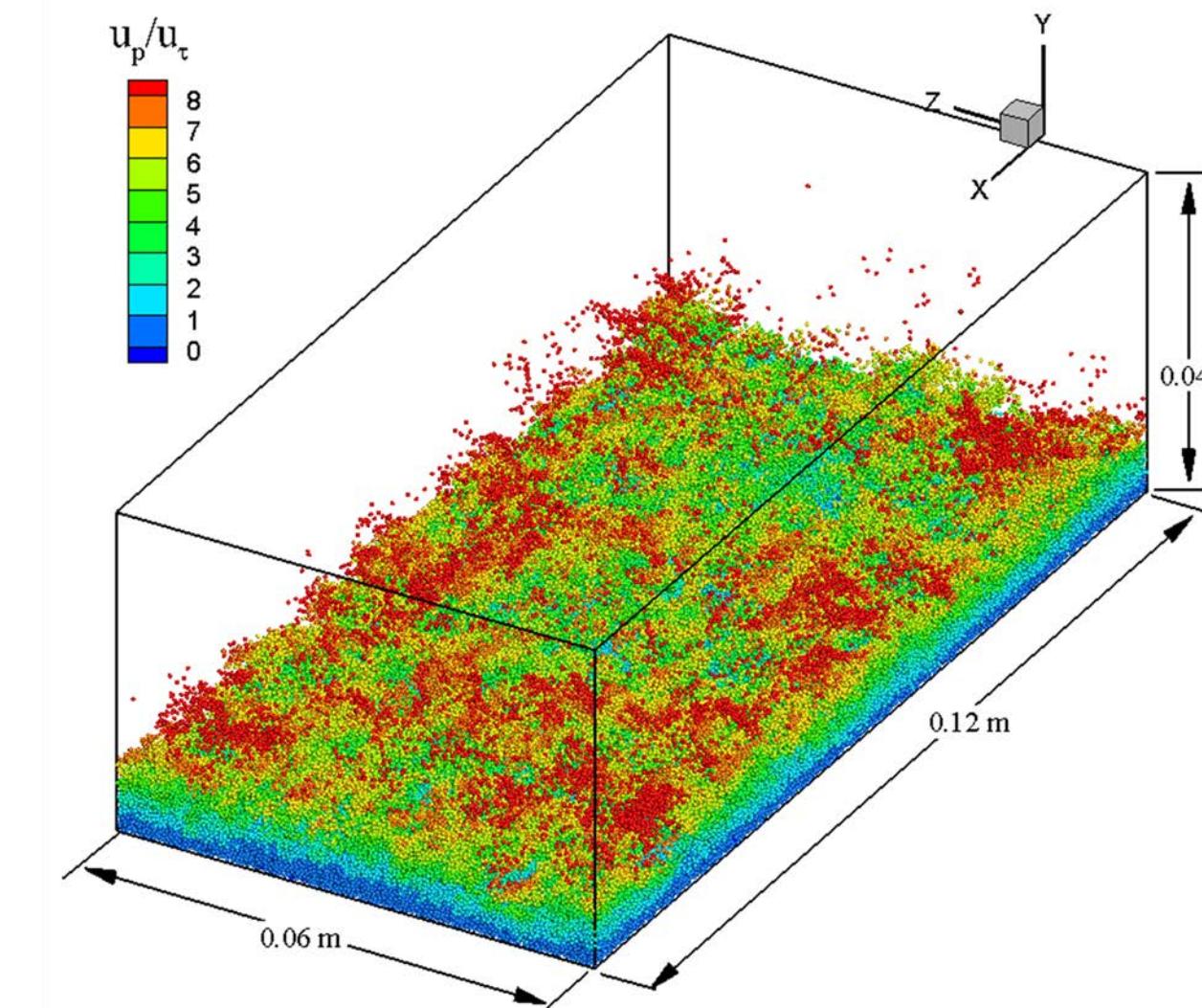


# Case-2: Turbulent open channel flow

- Schmeeckle 2014
  - Medium sand in water

	$u_\tau$	$Re_\tau$	$Re_b$	$\tau^*$	$u_\tau/\omega$
Run-1	0.013	584	7,820	0.0209	0.239
Run-2	0.0486	2170	22,286	0.288	0.848
Run-3	0.113	5080	53,116	1.558	1.558

Parameter	value
Particle diameter	0.5 mm
Coefficient of restitution	0.01
Particle density	2650 kg/m <sup>3</sup>
Friction Coefficient	0.6
Stiffness coefficient	100 (N/m)
Particle time step (10 sub steps)	5.3x10 <sup>-5</sup> s

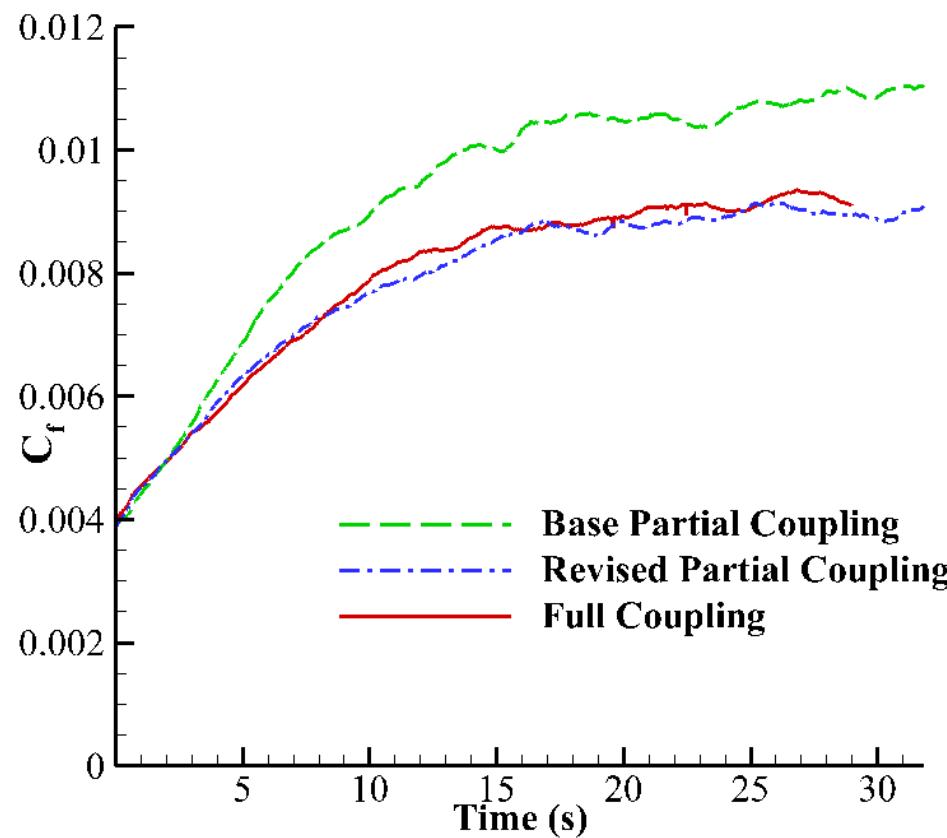


# Open channel: Averaged results

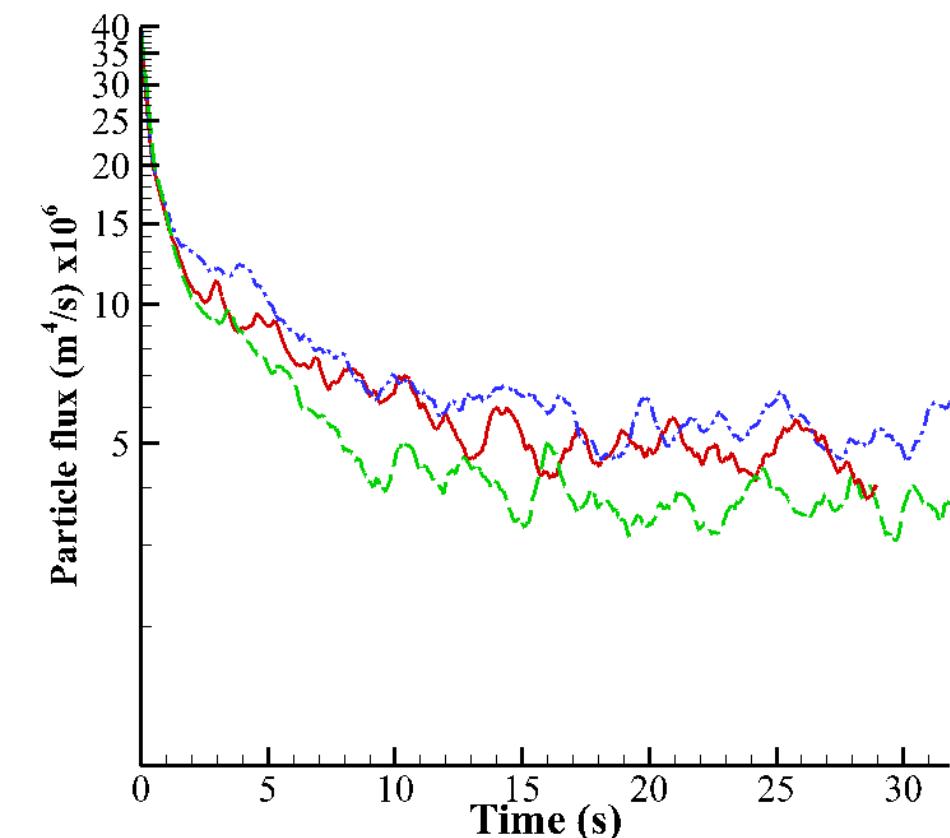
			Full coupling	Base particle coupling	Revised partial coupling
Run-1 $(\tau^* = 0.209)$	$C_f$	Mean	0.005582	0.006176	0.006096
		Diff. %	--	10.6%	9.2%
	Particle flux $(m^4/s)$	Mean	$1.5 \times 10^{-9}$	$1.14 \times 10^{-9}$	$1.26 \times 10^{-9}$
		Diff. %	--	-24.4%	-16.2%
Run-2 $(\tau^* = 0.288)$	$C_f$	Mean	0.009488	0.0096	0.009862
		Diff. %	--	1.2%	3.9%
	Particle flux $(m^4/s)$	Mean	$1.31 \times 10^{-7}$	$9.47 \times 10^{-8}$	$1.28 \times 10^{-7}$
		Diff. %	--	-27.8%	-2.2%
Run-3 $(\tau^* = 1.558)$	$C_f$	Mean	0.009151	0.010747	0.008956
		Diff. %	--	17.4%	-5.6%
	Particle flux $(m^4/s)$	Mean	$4.75 \times 10^{-6}$	$3.7 \times 10^{-6}$	$5.4 \times 10^{-6}$
		Diff. %	--	-21.5%	14.7%

# Open channel: Time series

Channel friction ( $C_f = (u_\tau/u_b)^2$ )



Sediment flux ( $m^4/s$ )



# Simulation speed-up

- Fluidized bed
  - overall speed-up ~27% (20% at stationary conditions)
  - time to fluid solution ~ 44% (41% at stationary conditions)
- Channel flow calculation (suspended load case)
  - overall speed-up~23%
  - time to fluid solution ~ 54%

# Conclusions

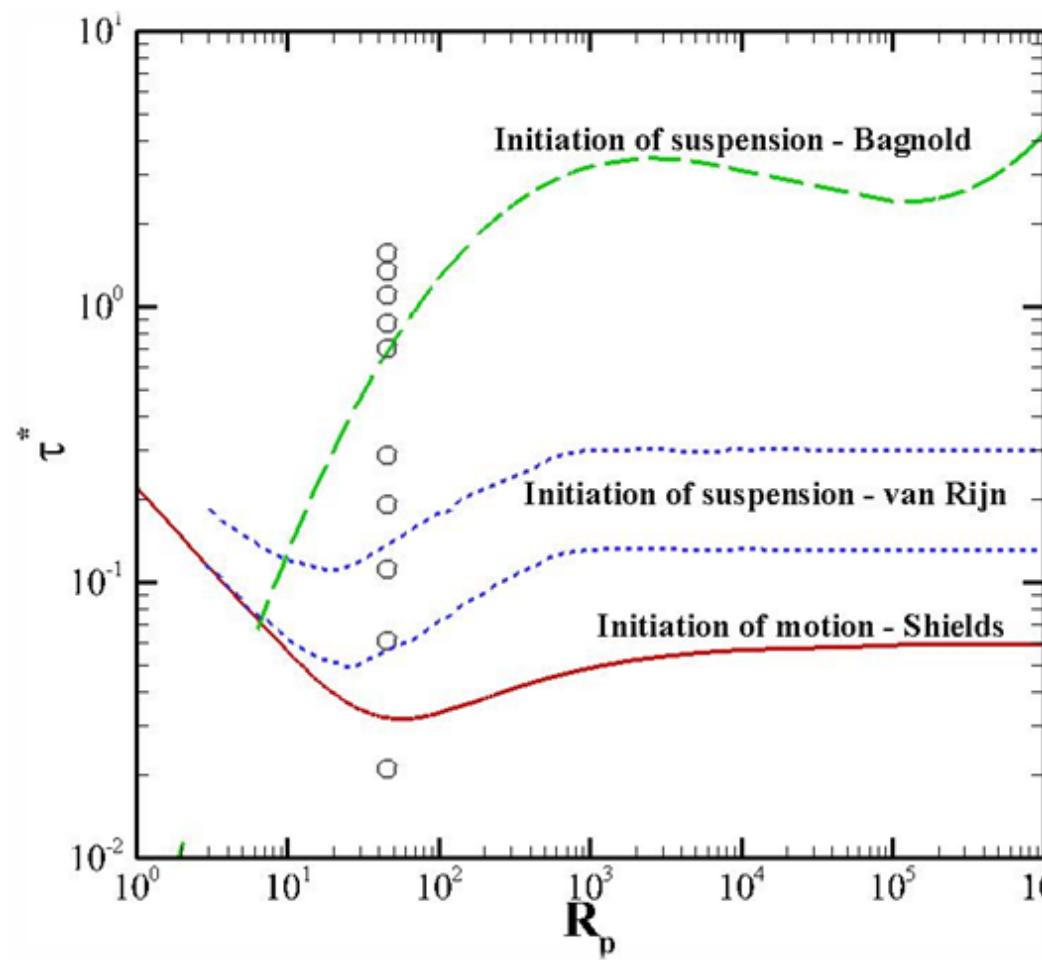
- Revised/improved partial coupling shows an improvement of the results of bubbling fluidized bed simulation and channel flow compared to base partial coupling
- >40% saving in solution time is achieved when using revised partial coupling as a result of better convergence behavior
- Other point force models can be treated similarly

# Acknowledgement

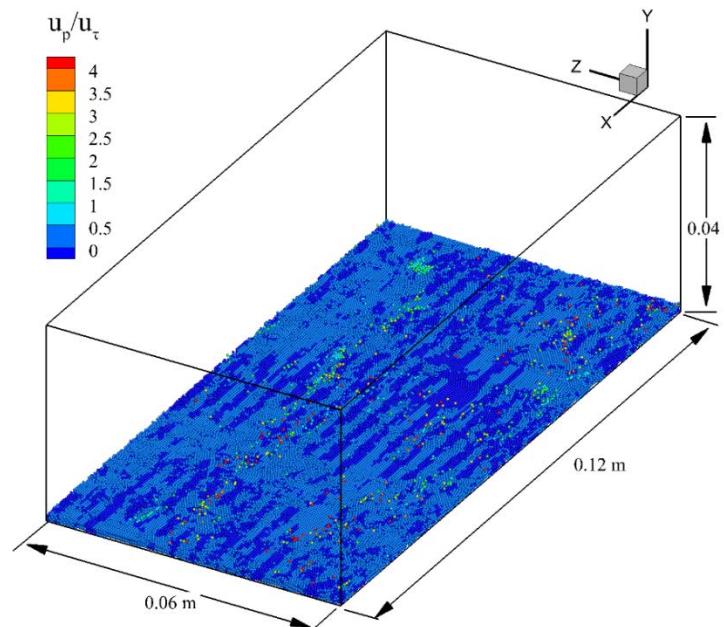
- Libyan Ministry of Higher Education and Scientific Research
- Department of Mechanical Engineering at Virginia Tech
- Advanced Research Computing (ARC) at Virginia Tech



*Thank You*

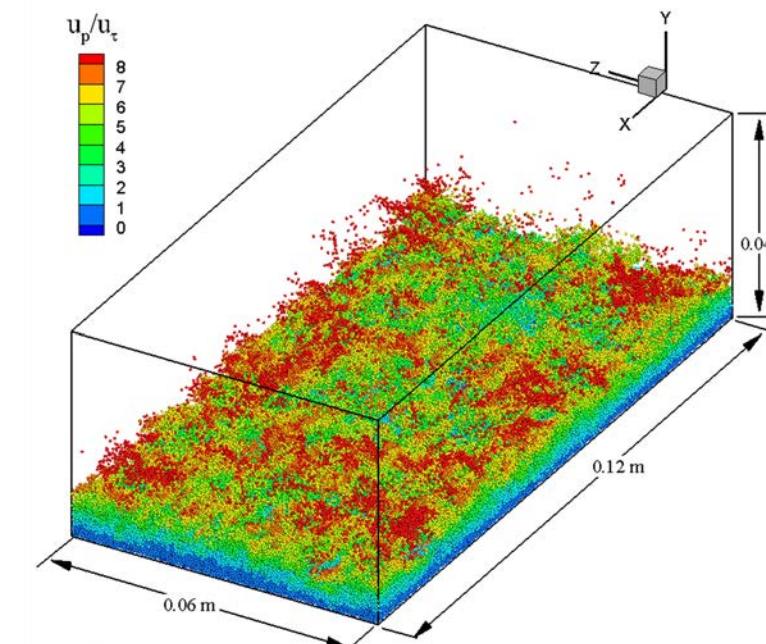
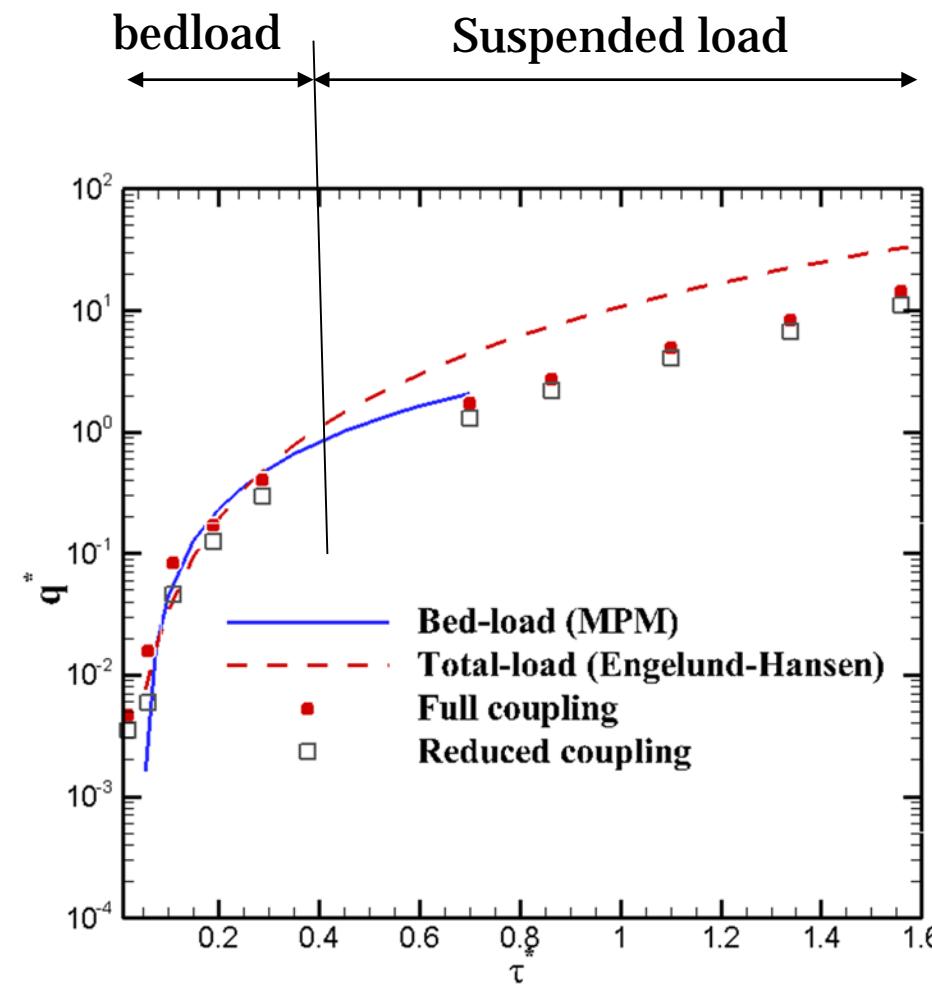


# Coupling effect in turbulent open channel flow



$$\text{Einstein number; } q^* \equiv \frac{q_{sx}}{\left( \left( \frac{\rho_s}{\rho} - 1 \right) g d^3 \right)^{1/2}}$$

$$\text{Shields parameter; } \tau^* \equiv \frac{\rho u_\tau^2}{(\rho_s - \rho) g d}$$



# Simulation speed-up: Fluidized bed

coupling	Time steps	Total time (hours)	Speed-up in total time	Press. Sol. Time (s)	Speed-up in Press. Sol. time
Full coupling	500k-init	59	NA	$0.4863 \times 10^4$	NA
Full coupling	500k-avg	64	NA	$0.5793 \times 10^4$	NA
Revised-PC	500k-init	39	-34%	$0.2541 \times 10^4$	-48%
Revised-PC	500k-avg	51:18	-20%	$0.3446 \times 10^4$	-41%
Base-PC	500k-init	51:30	-13%	$0.2170 \times 10^4$	-55.4%
Base-PC	500k-avg	94	47%	$0.4051 \times 10^4$	-30%

# Simulation speed-up: Open channel

Simulation times for 50k time steps (time 20-25s)

		Pressure conv. time (s)	Total time (s)	% DEM	Simulation speed-up	Pressure convergence speed-up
Run-1*	Full	1115.82	372500	65.7	NA	0.0
	PC-revised	132.734	275700	88.1	-26.0	-88.1
	PC-base	111.308	353000	88.3	-5.2	-90.0
Run-2*	Full	617.115	230900	67.2	NA	0.0
	PC-revised	106.584	131650	78.2	-43.0	-82.7
	PC-base	118.14425	245825	85.8	6.5	-80.9
Run-3	Full	215.731	187500	81.0	NA	0.0
	PC-revised	98.524	144800	78.0	-22.8	-54.3
	PC-base	100.729	254900	87.1	35.9	-53.3

\* Runs on different clusters